Detecting physical defects: A practical 2D-study of cracks and holes

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Abstract

Theoretical work on Qualitative Spatial Reasoning (QSR) is abundant, but the actual requirements of practical applications have been widely ignored. This paper discusses how ontologies allow to compare different QSR formalisms with respect to definability of spatial concepts, which are taken from a real-world problem. We introduce the problem of detecting physically defective parts (such as in manufacturing) and review which qualities are necessary for modeling these as QSR problem. We show that – besides standard mereotopological concepts – a set of artifacts, especially cracks and holes, are of foremost importance in the domain of interest. However, most currently available region-based QSR approaches fail to distinguish these. In the future, the proposed set of problem can be used to evaluate different QSR formalisms for their adequacy with respect to defining and distinguishing cracks and holes.

Introduction

For some decades now, qualitative reasoning approaches have been considered in the KR community to model space in different manners. Qualitative approaches use a more general version of scales than standard numerical approaches. Commonly, when one thinks about space, one uses a 'ratio' scale (Stevens 1946; Frank 2005) with an absolute 'null', so that notions as 'twice as big' are well-defined. On the contrary, qualitative spatial reasoning frameworks use more generic scales; usually either ordinal (elements are totally or at least partially ordered and such are comparable; e.g. 'BigRegion' $\geq$ 'SmallRegion', N, S, W, E, or the partial order defined by mereological subsumption) or nominal scales (only equivalence is defined, one cannot compare different units; e.g. connected, externally connected, disconnected). In QSR, qualities 'can only take a small, predetermined number of values' (de Kleer & Brown 1985). For a thoughtful introduction to the relevance of qualitative spatial reasoning, see (Frank 1996).

Natural languages and human cognition allow a broad range of different qualities; humans easily switch amongst them as necessary. However, for formal qualitative spatial representations, we need to specify these qualities accurately to understand which qualities are necessary and useful in different applications of spatial reasoning. So in order to evaluate different QSR frameworks, one needs to determine which qualities frameworks model and reason with. This amounts to the question of which qualitative spatial properties are definable within a particular formalism. For example region-based frameworks that restrict themselves to purely mereotopological concepts cannot answer any direction- or shape-based questions.

Evaluating definability using ontologies

One way of evaluating different QSR frameworks is through the use of ontologies. By an ontology we mean an axiomatic theory T_Σ written in a formal logic, usually first-order logic or a subset (or sometimes a superset such as second-order-logic) thereof. We prefer first-order logic over any higher-order logic to rely on their sound and complete proof system, so that every inference we can make can be proved from the axioms alone and vice verse. Ontologies are a powerful tool in evaluating the expressive power of different QSR formalisms. Assume $\mathfrak{M}^c_\Sigma$ and $\mathfrak{M}_\Sigma$ are two models of T_Σ, i.e. $\mathfrak{M}^c_\Sigma \models T_\Sigma$ and $\mathfrak{M}_\Sigma \models T_\Sigma$. Think of $\mathfrak{M}^c_\Sigma$ as a model of an ontology T_Σ whose real-world interpretation contains a crack (or cracks), then there must exist a sentence Φ_c s.t. $\mathfrak{M}^c_\Sigma \models \Phi_c$ but $\mathfrak{M}_\Sigma \not\models \Phi_c$ for any model $\mathfrak{M}_\Sigma$ of T_Σ without crack(s). We can distinguish (discriminate) two models $\mathfrak{M}^c_\Sigma$, $\mathfrak{M}_\Sigma$ in an ontology T if and only if such a sentence $\Phi_c \in \Sigma(T)$ exists.

The beauty of ontologies for evaluating definability is that most QSR formalisms are already formalized in an axiomatic way. Moreover, ontologies can simulate the behavior of QSR systems that are not fully described in an axiomatic way. So if a certain QSR implementation Σ returns the answer ϕ_Σ for a given input ψ_Σ, then the ontology simulating its behaviour should infer $\psi_\Sigma \cup T_\Sigma \models \phi_\Sigma$, i.e. ϕ is entailed by $T \cup \psi$.

Comparing definability of concepts amongst different QSR ontologies (and other QSR formalisms) is largely independent from evaluating reasoning aspects. Instead, it purely focuses on the expressiveness of ontologies of QSR frameworks, while leaving out more reasoning-centered questions such as reasoning complexity and optimization. Ultimately, this is equivalent to asking how appropriate a certain QSR formalism is for exchanging spatial information between different spatial systems in the sense of (Fonseca *et al.* 2002). Different systems usually use different underlying formalisms for reasoning with similar data, usually suited to a specific intended reasoning application. However, controlled exchange of spatial data among systems is impossible unless a mapping of their underlying representations or models is available. To come up with such a mapping, we first must understood the semantics of the spatial representations. In previous work (Hahmann & Gruninger 2008), we showed how this is possible using representation theorems that capture all models of a mereotopological ontology. Once the models of different qualitative spatial ontologies with similar concepts have been captured by equivalent, well-understood and well-classified mathematical structures, one can use the knowledge about the mathematical structures to construct a mapping between the ontologies. Moreover, such representation theorems establish the relation between the ontologies and exhibit which properties will be maintained or lost by mappings between certain ontologies.

Essentially, using ontologies as formal definition of models of QSR frameworks allows us to evaluate the expressiveness of the framework impartial of properties relevant for reasoning within the QSR frameworks. As a welcomed side-effect known algorithms from the representation domain can be directly applied to the spatial models themselves, with the hope of providing efficient algorithms. Finally, models from the representing class of mathematical structures can be verified against the spatial domain of interest to check the adequacy of a QSR framework.

In the next section, we briefly explain the qualities applicable to our domain of manufacturing metal sheets and show how they generalize to similar applications. Afterwards, we introduce the example domain and demonstrate what kind of differences in models one might want to define within qualitative spatial ontologies.

Qualities in QSR

It is essential to classify all qualities that the QSR community is interested in. Here we just give a list of qualities we consider relevant for our practical application discussed later in the paper. We do not claim completeness of the list for all purposes of QSR, although we list the spatial qualities that we think are sufficient to capture most practical domains and are relevant to region-based QSR. If we extend this scope, a larger set of qualities is definable.

- Topology (incl. different types of connection)
- Mereology (Parthood, Overlap)
- Morphology (Shape-related properties such as convexity/concavity, curvature, corners, notions of congruence)
- Dimension (e.g. frameworks might explicitly distinguish spatial entities of different dimensions, restrict itself to entities of same dimensions, or make no assumption about dimension at all)
- Direction, Orientation (North, East, South, or West; right or left; below or on top; inside or outside; parallel, orthogonal or in between)
- Qualitative size and distances
- Fuzzy qualities (vague, uncertain, or approximate qualities; not further discussed in this paper)

An overview (though also not complete) of different qualitative spatial reasoning frameworks and the qualities they model can be found in (Cohn & Hazarika 2001), which further explores the boundary between qualitative and quantitative properties. Next, we briefly explain some of these qualities and link to relevant work.

Topology, Mereology, and Mereotopology The integration of mereological (i.e. of parthood) and topological (i.e. of connectedness) qualities have been widely considered, for an overview see (Varzi 1996; Casati & Varzi 1999). Topology alone is well-understood due to a long interest in mathematics; for spatial reasoning it has been considered in (Egenhofer 1991). Mereology is also well-studied, for an comprehensive overview see (Simons 1987). Moreover, the problems arising from holes (in the larger sense, i.e. including depressions, hollows, etc.) in mereotopology have already been researched - primarily from the philosophical perspective (Casati & Varzi 1994). Topology together with holes has been considered by (Egenhofer, Clementini, & Di Felice 1994).

Morphology Morphology has been addressed in some QSR frameworks, most notably in (Borgo, Guarino, & Masolo 1996) and in spherical approaches to mereotopology (Tarski 1956; Bennett *et al.* 2000; Bennett 2001). However, especially the spherical approach lacks any morphological quality beyond congruence. Other promising shape-based approaches can be found in (Gruninger 2000), which gives an axiomatic theory of shapes with straight edges (polygons) building on Hilbert's geometry. (Pratt & Schoop 1997) also give a mereotopological theory in which each valid model can be interpreted as a set of polygonal regions in the two-dimensional closed plane. Either work considers shapes that are specified only qualitatively and thus either one provides a solid foundation for qualitative representations of space. However, extensions with other qualities such as convexity or curvature, are desirable.

Dimension Most applications of QSR frameworks need only to consider a maximum of three dimensions with potentially an additional temporal component. Hence, it might be useful to refrain from formalisms which give up expressiveness by abstracting from the dimension of regions. Considering formalisms that explicitly model regions of different dimensions as different classes of objects seem more adequate in capturing the world around us. Further restricting ourselves to maximal two dimensions is a feasible way to deal with some practical applications in order to reduce complexity and accommodate the fact that common computer vision systems are also limited to capture two-dimensional images.

Qualitative direction, orientation, size, and distances (Freksa 1992) provides an early overview of orientation qualities used within QSR, but little work has been done on orientation properties in region-based QSR frameworks. The problem of distinguishing interiors and exteriors of regions has been addressed in (Gruninger 2000) in the context of object recognition. (Moratz, Renz, & Wolter 2000; Schlieder 1995) use directional approaches for line segments, but we are not aware of a combination with region-based approaches. However, a framework using topological and directional qualities has been presented in (Sharma 1996). Distances have been considered qualitatively, e.g. in (Hernández, Clementini, & Di Felice 1995). Recently, (Dong 2008) showed how *RCC++*, an extension of *RCC*, can be used to introduce concepts of distance and size into the formerly strictly mereotopological framework.

Evaluation of qualities in QSR formalisms

Apart from evaluating definability of concepts in QSR frameworks it seems in general unrealistic to compare frameworks modeling different qualities directly amongst each other. A more realistic approach is to identify practical models and problems (queries we want to answer) and investigate: (1) what qualities they require, (2) how adequately the frameworks capture these qualities, (3) whether the QSR frameworks are capable of answering the queries, and (4) what is the complexity of answering these queries. So far, the last point has received some attention, e.g. in (Renz & Nebel 1999; 2007), although in a more general way of looking at the complexity of the whole reasoning framework *RCC* and tractable subsets thereof. However, without further model constraints these results apply to the complexity of answering queries expressable in the *RCC*. In the remainder of this paper, we focus on question (1) by introducing artifacts occurring in manufacturing processes which most current frameworks cannot define.

Exemplary application of region-based QSR: Detecting physically defective parts

Now we introduce a problem domain where qualitative spatial ontologies (in particular region-based ones) can be used for defining a set of physical defects. The examples are taken from manufacturing of products, where parts are molded, cut, and joined. In such a setting, automatic supervision of the production process requires the identification of parts that deviate from the product, in order to maintain production of high quality. One criteria of detecting defective parts or products is by its shape and the way individual parts are assembled. Any QSR that is of practical use in our domain, needs to be able to distinguish wanted from unwanted physical defects by their shape, location, and connection properties. For example, an automated supervision of the process should detect cracks, unwanted holes, dents or similar deviations[1] in the parts. Moreover, it should recognize connectedness of parts throughout the stage, to avoid products containing broken (in the physical sense) parts. Finally, one might want to detect the right type of 'connectors' in order to connect two parts properly (think of puzzle elements, cable-connectors, or 'key and slot joints')[2]. Therefore, the following concepts need to be definable in any ontology or QSR formalism useful for our domain.

[1] So-called superficial discontinuities (on surfaces in any dimension), see (Casati & Varzi 1994) for a detailed discussion.

[2] See (Kim, Yang, & Kim 2008) for an engineering perspective of using mereotopology to model different kind of joints.

(a) Missing point (b) 'Positive' crack

Figure 1: Two models contrary to common-sense

(a) Model that contains two ridges enclosing an hole, one holed ridge, or two grooves (at the side) and a hole.

(b) Model that contains one branching, two (one bending), or three cracks depending on the perception.

Figure 2: 2D-configurations with multiple interpretations

Dimension

Real-world reasoning problems are usually restricted to two- and three-dimensional objects, apart from temporal aspects. Since little seems to be known about application domains, we decided to restrict ourselves here to two-dimensional problems, simplifying the domain and our discussions. Such two-dimensional models are frequently used in the production of metal sheets where thickness is irrelevant. As a common day example, paper-cutting processes can be considered similarly. Most of the ideas we present here can be extended to three-dimensional problems by introducing surfaces and faces as additional two-dimensional artifacts.

Defects (artifacts) in two-dimensional physical objects:

- 0-dimensional: missing points
- 1-dimensional: cracks
- 2-dimensional: holes (in the most general sense, including grooves, ridges, depressions, hollows, etc.)

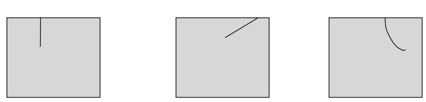
Artifacts

We intend to model significant, actually occurring artifacts in two-dimensional objects. We do not intend to capture all theoretically possible artifacts, but instead want to give a naïve understanding of common artifacts occurring in the domain. To simplify things, we consider artifacts of each dimension individually. Missing points (zero dimensions) are very difficult to conceive in the real-world, since they are always perceived as crack or hole (of some extent). Both cracks and holes are theoretically possible in the positive as well as negative sense: proper holes are those in the interior of some region; however, ridges (a type of negative holes, or the counterpart to a groove) add something to the region itself. When considering cracks, theoretically both positive and negative types can occur: proper cracks dividing a regions into two (usually modeled as externally connected regions), but also one-dimensional objects attached to a region: a 'positive crack' that adds to the region (see the *RCC* example discussed in (Dong 2008)). However, the later is implausible in the real world unless it is a crack of the surrounding object, but then it is addressed as proper crack. So we focus on proper cracks, proper holes, and grooves since these are sufficient to model all superficialities in two-dimensional regions.

Combinations of cracks and holes are possible. e.g. two holes (or one disconnected hole) can exist, but it is also possible that there are both cracks and holes (connected or not) in a certain spatial configuration. Connection (including external connection) as well as parthood and overlap properties deal with that. Distinguishing or counting holes and other artifacts is not trivial, see (Casati & Varzi 2004) for an interesting reflection on this issue. To avoid these tricky issues, we focus on self-connected artifacts that are either of dimension one or two. Now we take a closer look at their morphological and orientation properties.

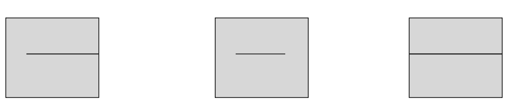
Cracks Cracks as one-dimensional artifacts have no specific shape apart from their curvature or potential corners within the crack (could also be modeled by a set of connected cracks). However, topological and orientational qualities capturing the relative position to the host can be of importance. We can distinguish tangential, non-tangential, and separating cracks, where the later seems equivalent to two externally connected regions, and we can capture perpendicularity or parallelism of straight cracks relative to the hosting region or to each other.

- Topology (position relative to host (truly tangential, interior, separating), compare figure 4)
- Morphology (curvature, compare figures 3a and 3b to 3c, congruence)
- Direction, Orientation (relative to hosting body, e.g. perpendicular in figure 3a vs. non-perpendicular in figure 3b, as well to each other (parallel or perpendicular))



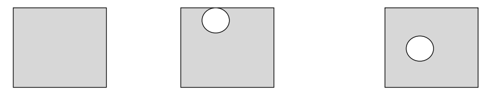
(a) Straight, perpendicular (b) Straight, diagonal (c) Curved

Figure 3: Curvature and perpendicularity of cracks



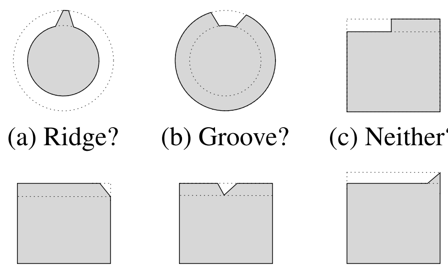
(a) Border crack (b) Interior crack (c) Full crack

Figure 4: Relative position of cracks to their host



(a) Without hole (b) Tangential hole (c) Non-tangential hole

Figure 5: Relative position of holes to their host



(a) Ridge? (b) Groove? (c) Neither?

(d) Groove or not? (e) Groove or two ridges? (f) Ridge or not?

Figure 6: Superficialities: problem of distinguishing ridges and grooves (grey region in consideration)

Proper holes (Cavities) Almost all properties that are used to describe regions qualitatively also apply to holes. In particular the topological, mereological, and morphological ('gestalt') properties are equivalent. However, contrary to (positive) regions, holes (negative regions) cannot have any crack artifacts themselves - for the same reason that regions cannot have 'positive' cracks. However, they are allowed to contain any kind of ridge or groove with the difference that the space is inverted: a hole with a groove adds space to the host object whereas a hole with a ridge reduces the space occupied by the host object. In addition to these properties, only the relative (topological) position of a proper hole to its host seems practically relevant, see figure 5. Besides that, mereotopological relations between disconnected proper holes, or parts of holes are relevant, but not discussed in detail here. Nevertheless, any region-based QSR formalism should be able to define models with a single connected hole and distinguish them from otherwise similar models with disconnected holes.

Grooves and Ridges (Superficialities) Grooves are handled in (Casati & Varzi 1994) just as special kind of holes[3]. These are not just tangential, but the hole properly overlaps the host region, so that there is an opening. Based on convexity, curvature, and existence of corners, we can distinguish a multitude of grooves that occur in molding and joining processes. There is an intrinsic difficulty in distinguishing grooves and ridges, since they are interchangeable. In particular, any groove can also be regarded as one or more ridges (depending on the shape) and vice verse, see Figure 6

[3](Casati & Varzi 1994) show how holes and other 'negative' discontinuities such as grooves or depressions can always be ascribed to a concavity in the surface; however, concavity is not a sufficient condition for their existence.

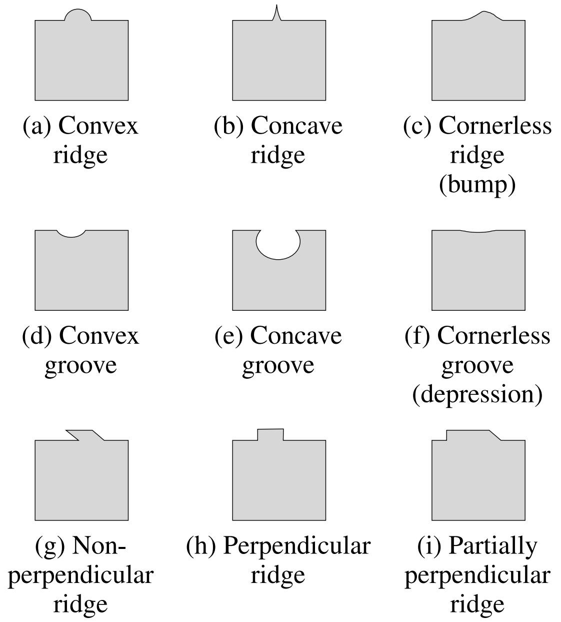


Figure 7: Superficialities: convexity, curvature, and existence of corners (grey region in consideration)

for examples. Amongst other background knowledge, such as the functional relationship to the host object, humans conceive these superficialities as grooves or ridges depending on the relative size to the host object. A small missing piece (as in Figures 6b, and 6e) is usually seen as a groove and not as a large ridge. However, if both are about the same size, it is usually neither called a groove nor a ridge. Notice that because a groove is a negative region whereas a ridge is positive, the distinction between them also matters when describing their curvature: one artifact described as convex groove corresponds to one or multiple concave ridge(s). Figure 7 gives an overview of the morphological qualities relevant to model ridges and grooves. The properties in 7 (g)-(i) apply to both ridges and grooves.

Inadequate QSR formalisms

If we want to detect the described artifacts with a QSR framework, a region-based QSR formalism seems most adequate[4]. In order to define and distinguish cracks and holes, the formalism then needs to focus on the following properties: connectedness (in particular self-connectedness), parthood (special parthood relations might be used to identify holes or branching cracks) regularity (to identify cracks), convexity, and curvature (to identify unwanted dents). Common QSR frameworks only address a subset of these properties, but might still be useful to distinguish some of the configurations we mentioned. For example without ways to capture convexity and/or curvature, one will not be able to identify holes, or grooves in the shape. However, one could still recognize cracks or broken parts.

Clarke (Clarke 1981), the *RCC* (Cohn *et al.* 1997), and the system *RT* (Asher & Vieu 1995) all accommodate similar mereotopological concepts and need to be evaluated with respect to whether they can detect holes (and maybe distinguish allowed ones from unwanted ones). Certainly, they cannot recognize any-shape based defects such as dents, bumps, or corners cut-offs. Moreover, (Dong 2008) claims that the connection relation in *RCC* cannot accommodate any notion of size or distance without major changes to the theory (the claim extends to the theories RT and that of Clarke). However, we might be able to *add* a separate notion of distance to increase the expressiveness of the frameworks. The system of Borgo, Guarino, and Masolo (Borgo, Guarino, & Masolo 1996) incorporates the notion of convexity addressing some of these problems, but still seems too weak to distinguish some of the example models. (Pratt 1999) explains how mereological QSR approaches extended with a notion of convexity can be seen as similar to affine geometry (and thus more generally to projective geometry), a quite expressive, but still qualitative modeling of space.

The set of examples demonstrates how different settings can be used to evaluate whether known qualitative spatial formalism can distinguish certain configurations. For example in the frameworks *RCC* and *RT*, a crack (or cut) from the boundary to the middle of some region will not be distinguished from a region with an interior crack. Both cannot occur in models of *RT* and *RCC*, so the axiomatic theories are too weak for distinguishing these configurations. However, we need to understand that it does matter less which qualities a formalism uses, but more whether the formalism is capable of defining certain spatial concepts accurately and therefore can detect and distinguish them.

Summary and Outlook

Finding real-world problems is an important issue in the QSR community to evaluate QSR formalisms. Taking ideas from previous work on holes (Casati & Varzi 1994), we collected two-dimensional configurations relevant to manufacturing. These can serve as benchmarks for comparing the definability of holes and cracks in QSR formalisms. Defining these artifacts accurately requires a wide range of qualities that we identified. Of course, it is arguable how far qualitative reasoning goes and how to separate it from the grey zone in between qualitative and quantitative approaches. Nevertheless, to apply QSR to real problems, one needs to extend the simplistic properties of current region-based QSR ontologies. We are unaware of any QSR ontology in which the concepts of cracks and holes are definable in regions of two or more dimensions. To different degrees, most region-based formalisms are limited to mereotopological features; few of them are extended by fairly general morphological distinctions (convexity, congruence). Especially the common assumption of regular regions contradicts our findings that in manufacturing it is desirable to model artifacts of one dimension lower than the actual objects in mind. New, more expressive QSR formalisms are necessary. Developing first-order ontologies that address these problems will be part of our future research.

[4]Other non-region-based approaches, e.g. using line segments, or projective, affine, or incidence geometry (Balbiani *et al.* 2007) might be useful either by themselves or by supplementing region-based approaches.

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